# Philosophy of Mathematics Fall 2024

### **PHIL 257**

Published Sep 04, 2024

#### **Class Schedule**

Course	Meet Days	Meet Time	Location	Instructor(s)
PHIL 257 001 [LEC]	Tue, Thu Sep 4 - Dec 3	02:30PM - 03:50PM	AL 124	P. Marino pmarino@uwaterloo.ca (mailto:p

schedule data automatically refreshed daily

### Instructor & TA (Teaching Assistant) Information

Instructor: Patricia Marino

Office: HH 329

Office Hours: Mondays 3:30-4:30 and by appointment, in person or virtual. Subject to change so check Learn or email me.

Email: pmarino@uwaterloo.ca

### **Course Description**

Calendar Description for PHIL 257:

An introduction to philosophical problems concerning mathematics. Topics may include: what makes mathematical statements true (e.g., do numbers exist?); whether mathematics is a human creation or something we discover; what counts as a proof; mathematical paradoxes; the relationship between mathematics and other sciences; and mathematical pluralism.

View requirements for PHIL 257 (https://acal.fast.uwaterloo.ca/course/1249/PHIL/257)

Philosophy of mathematics considers questions about mathematical objects, truth, and methodology. What are numbers? Is mathematics is a human creation or something we discover? What are the proper methods of reasoning in mathematics, and what makes them the right ones? Is there something miraculous about the way math is applicable in science? By engaging with readings, lectures, and class discussions, students in this course will learn why these questions have sparked a wide range of answers. Students will learn to formulate their own ideas about these questions and will learn to communicate those ideas orally and in writing.

## **Learning Outcomes**

#### By the end of this course students should be able to:

give examples of disputed topics in mathematics and philosophy of mathematics

analyze and critically evaluate divergent views in mathematical foundations and practice

develop and express their own views about mathematics and philosophy of mathematics

effectively communicate their ideas orally and in writing

### **Tentative Course Schedule**

## Week 1: Introduction

• Thursday, September 5: no reading assigned

Week 2: Philosophical questions: what is math, and why is it useful? What are numbers, and what makes mathematical statements true?

- Tuesday September 10: Eugene Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," Richard Courant Lecture in Mathematical Sciences delivered at New York University, May 11, 1959, Communications on Pure and Applied Mathematics 8 (1960).
- Thursday September 12: Gottlob Frege, selection from Selection from Foundations of Arithmetic (Introduction, sections 5-8, sections 34-38).

## Week 3: Frege's logicism and the implications of Russell's Paradox: is mathematics just logic?

- Tuesday September 17: Gottlob Frege, selection from Foundations of Arithmetic, (sections 45-83).
- Thursday September 19: Andrew Irvine, selection from, "Russell's Paradox" at the Stanford Encyclopedia of Philosophy. Read the section "History of the Paradox."

#### Week 4: Hilbert's formalism and the implications of Gödel's Incompleteness Theorem: is mathematics about manipulating symbols?

- Tuesday September 24: David Hilbert, "On the Infinite," from Benacerraf and Putnam, Philosophy of Mathematics, Selected Readings, 2nd edition (Cambridge University Press, 1984) pp. 183-200, this translates the originally published 1926 article "Über das Unendliche," Mathematische Annalen, 95: 161-90.
- Thursday September 26: Solomon Feferman, "The Nature and Significance of Gödel's Incompleteness Theorems."

## Week 5: Heyting's intuitionism and a shift away from classical logic: are mathematical objects mental constructions?

- Tuesday October 1: Arend Heyting, The Intuitionistic Foundations of Mathematics," reprinted in Benacerraf and Putnam, *Philosophy of Mathematics, Selected Readings*, 2nd edition (Cambridge University Press, 1984).
- Thursday October 3: Arend Heyting "Disputation" reprinted in Benacerraf and Putnam, *Philosophy of Mathematics, Selected Readings*, 2nd edition (Cambridge University Press, 1984). **First paper due.**

### Week 6: The problems of mathematical truth and existence: why is there so much disagreement?

- Tuesday October 8: Paul Benacerraf, "Mathematical Truth," The Journal of Philosophy, 70 (1973), 661 679.
- Thursday October 10: FIRST IN-CLASS TEST

### --reading week---

#### Week 7: Fictionalism: are mathematical statements not true after all?

- Tuesday October 22: Stephen Yablo, "The Myth of the Seven," Fictionalism in Metaphysics (Oxford University Press, 2005).
- Thursday October 24: John Burgess, "Mathematics and Bleak House," Philosophia Mathematica, 12 (2004), 18-36.

### Week 8: Naturalism: is mathematics necessary for science, and if it is, does that mean it must be true?

- Tuesday October 29: Penelope Maddy, "Indispensability and Practice." The Journal of Philosophy 89, no. 6 (1992): 275-289.
- Thursday October 31: Penelope Maddy, "Three Forms of Naturalism." Oxford Handbook of Philosophy of Mathematics and Logic.

### Week 9: Formal and informal proofs: what constitutes mathematical justification?

- Tuesday Nov 5: Fenner Tanswell, "A Problem with the Dependence of Informal Proofs on Formal Proofs." *Philosophia Mathematica* 23, no. 3 (2015): 295–310.
- Thursday Nov 7: Silvia De Toffoli, "Groundwork for a Fallibilist Account of Mathematics." The Philosophical Quarterly 71, no. 4 (2021).

## Week 10: Critical math kinds: can there be alternative mathematics?

- Tuesday Nov 12: Franci Mangraviti, "Critical Math Kinds: A Framework for the Philosophy of Alternative Mathematics." Erkenntnis (2024): 1-21.
- Thursday Nov 14 CLASS CANCELLED and second paper due.

## Week 11: Pedagogy and indigenization: how should mathematics education respond to the Calls to Action?

 Tuesday Nov 19: readings TBA Thursday Nov 21: readings TBA

### Week 12: Revisiting the usefulness of mathematics: is the success of mathematics in science a miracle?

- Tuesday Nov 26: Sorin Bangu, "Wigner's Puzzle for Mathematical Naturalism." International Studies in the Philosophy of Science 23, no. 3 (2009): 245-263.
- Thursday Nov 28: Arezoo Islami, "A Match Not Made in Heaven: on the Applicability of Mathematics in Physics." Synthese 194, no. 12 (2017): 4839-4861.

### Week 13: Second test

## Tuesday Dec 3: SECOND IN-CLASS TEST

#### Texts / Materials

Title / Name	Notes / Comments	Required
Readings	Can be downloaded through the UW library via links provided or by dowloading PDFs from this Learn site under "Readings"	Yes

#### **Student Assessment**

Component	Value
Paper 1	15%
In-class test 1	20%
Paper 2	20%
In class test 2	20%
Final exam	20%
Attendance and Participation	5%

Both papers should be 900-1200 words and topics will be handed out. For the first paper, you have the option of handing in a rewrite based on my comments. If you choose to do this your new paper grade will be an average of the original and the rewrite; your grade will not go down if the new draft is worse, but improved grades require significant changes and not just small edits. For the second paper, I will write fewer comments; if you would like more feedback, don't hesitate to ask. Paper assignments will ask you to write about your own ideas about a philosophical problem while engaging the texts and ideas we've encountered in class. The focus is on presenting an original argument. Of course, this means the ideas in your papers must be your own; we will talk more in class about how to ensure that the ideas you present as your own really are, and how to cite any outside sources you do use appropriately. We will also discuss in class the use of generative AI and how it can and should be used. If you have any questions at any time about academic honesty and what it requires, do not hesitate to ask.

In class tests will be a mix of question formats including multiple choice and short answer. Test 1 covers the first half of the course and test 2 the second half. The final exam will include long-answer and essay questions.

For a course such as this, attendance and participation are important. There are several ways to participate in this class: you can participate in class discussion, or by contributing to discussion on LEARN, or by emailing or speaking with me personally. Attendance also counts toward your participation grade. If you attend class regularly, that will earn a grade of 65 for participation. If you participate in any of the various forms at least once per week, that will earn a grade at least in the 70s range. More frequent participation will raise your participation grade. I will post participation grades on LEARN at the end of term; if you don't agree with yours please email me and we can discuss it.

Obviously if you feel unwell you should stay home. I also know that students may encounter other obstacles to attending class. You may miss up a few class meetings in the term for any reason, but if you find that you will have to miss more meetings, please let me know and we will work something out. I will use LEARN for announcements etc. so please check it regularly.

# **Assignment Screening**

No assignment screening will be used in this class.

## **Administrative Policy**

## **Territorial Acknowledgement**

The University of Waterloo acknowledges that much of our work takes place on the traditional territory of the Neutral, Anishinaabeg, and Haudenosaunee peoples. Our main campus is situated on the Haldimand Tract, the land granted to the Six Nations that includes six miles on each side of the Grand River. Our active work toward reconciliation takes place across our campuses through research, learning, teaching, and community building, and is centralized within the

Office of Indigenous Relations (https://uwaterloo.ca/indigenous) .

## Pronouns and chosen or preferred first name

Students can verify their chosen or preferred first name by logging into <u>WatIAM (https://lidm.uwaterloo.ca/watiam/)</u>. Your chosen or preferred first name listed in WatIAM will be used across campus (e.g., in LEARN, Quest, WaterlooWorks, WatCard, etc). Your legal first name will always be used on certain official documents, such as diplomas and transcripts.

Students can view and change their pronouns in <a href="Quest">Quest</a> (<a href="https://uwaterloo.ca/quest/help/students/how-do-i/view-or-update-my-personal-information">https://uwaterloo.ca/quest/help/students/how-do-i/view-or-update-my-personal-information</a>). Should you choose to include your pronouns in Quest, your pronouns will be accessible by instructors, advisors, and employees with access to Quest. If no pronoun is added, the field in Quest will remain blank.

### Generative Al

Generative artificial intelligence (GenAl) trained using large language models (LLM) or other methods to produce text, images, music, or code, like Chat GPT, DALL-E, or GitHub CoPilot, may be used in this course with proper documentation, citation, and acknowledgement. Permitted uses of and expectations for using GenAl will be discussed in class and outlined on assignment instructions.

Recommendations for how to cite generative Al in student work at the University of Waterloo may be found through the Library: <a href="https://subjectguides.uwaterloo.ca/chatgpt\_generative\_ai">https://subjectguides.uwaterloo.ca/chatgpt\_generative\_ai</a> (https://subjectguides.uwaterloo.ca/chatgpt\_generative\_ai). Please be aware that generative Al is known to falsify references to other work and may fabricate facts and inaccurately express ideas. GenAl generates content based on the input of other human authors and may therefore contain inaccuracies or reflect biases.

In addition, you should be aware that the legal/copyright status of generative AI inputs and outputs is unclear. Exercise caution when using large portions of content from AI sources, especially images. More information is available from the Copyright Advisory Committee: <a href="https://uwaterloo.ca/copyright-at-waterloo/teaching/generative-artificial-intelligence">https://uwaterloo.ca/copyright-at-waterloo/teaching/generative-artificial-intelligence</a> (https://uwaterloo.ca/copyright-at-waterloo/teaching/generative-artificial-intelligence)

You are accountable for the content and accuracy of all work you submit in this class, including any supported by generative AI.

## **Mental Health Support**

All of us need a support system. The faculty and staff in Arts encourage students to seek out mental health support if they are needed.

#### On campus supports

All of us need a support system. The faculty and staff in Arts encourage students to seek out mental health and counselling if they are needed. For counselling (individual or group) reach out to <a href="Campus Wellness and Counselling Services">Campus Wellness and Counselling Services</a>. (https://uwaterloo.ca/campus-wellness/counselling-services). Counselling Services strives to provide a secure, supportive environment for students of all orientations and backgrounds. They offer confidential counselling for a variety of areas including anxiety, stress management, depression, grief, substance use, sexuality, relationship issues, and much more.

### Other on-campus supports

- MATES (https://wusa.ca/services/uw-mates): one-to-one peer support program offered by the Waterloo Undergraduate Student Association (WUSA) and Counselling Services
- Download the WatSafe app (https://uwaterloo.ca/watsafe/) to your phone to quickly access mental health support information
- Empower Me (https://uwaterloo.us11.list-manage.com/track/click?u=f5194df42b6fc63ac2f0875eb&id=6859501915&e=d4e4205e24): to access create an account on the Dialogue mobile app or on the web at <a href="https://www.studentcare.ca/dialogue">www.studentcare.ca/dialogue</a> (http://www.studentcare.ca/dialogue)
- Sexual Violence Prevention and Response Centre (https://uwaterloo.ca/sexual-violence-prevention-response-office/) supports all members of the University of Waterloo campus community who have experienced or been impacted by sexual violence.

## Off campus supports

- Good2Talk (https://good2talk.ca/): Free confidential help line for post-secondary students. Phone: 1-866-925-5454
- Grand River Hospital: Emergency care for mental health crisis. Phone: 519-749-4300
- St. Mary's Hospital (https://uwaterloo.us11.list-manage.com/track/click?u=f5194df42b6fc63ac2f0875eb&id=f9ffc07602&e=d4e4205e24) 519-744-3311
- Here 24/7 (https://here247.ca/) : Mental Health and Crisis Service Team. Phone: 1-844-437-3247
- 988 Suicide Crisis Helpline: a 24/7 urgent mental health care three-digit helpline
- OK2BME (https://ok2bme.ca/): set of support services for Two-Spirit, lesbian, gay, bisexual, trans, queer, intersex, asexual (@SLGBTQIA+) and questioning kids, teens, adults, and their families in Waterloo Region. Phone: 519-884-0000 or e-mail ok2bme@caminowellbeing.ca (mailto:ok2bme@caminowellbeing.ca)
- <u>Sexual Assault Support Centre of Waterloo Region (http://www.sascwr.org/)</u> offers individual counselling to people of all genders, 16+ who have been sexually assaulted at any point in their lives. As well as short-term support for family members or friends of sexual assault survivors. Also, information and referrals

More information about resources and supports for students can be found online in on the <u>Faculty of Arts Student Support</u> (<a href="https://uwaterloo.ca/arts/undergraduate/student-support/">https://uwaterloo.ca/arts/undergraduate/student-support/</a>) page.

# **Intellectual Property**

Students should be aware that this course contains the intellectual property of their instructor. TA, and/or the University of Waterloo.

Intellectual property includes items such as:

- Lecture content, spoken and written (and any audio/video recording thereof);
- · Lecture handouts, presentations, and other materials prepared for the course (e.g., PowerPoint slides);
- · Questions or solution sets from various types of assessments (e.g., assignments, quizzes, tests, final exams); and
- Work protected by copyright (e.g., any work authored by the instructor or TA or used by the instructor or TA with permission of the copyright owner).

Course materials and the intellectual property contained therein, are used to enhance a student's educational experience. However, sharing this intellectual property without the intellectual property owner's permission is a violation of intellectual property rights. For this reason, it is necessary to ask the instructor, TA and/or the University of Waterloo for permission before uploading and sharing the intellectual property of others online (e.g., to an online repository).

Permission from an instructor, TA or the University is also necessary before sharing the intellectual property of others from completed courses with students taking the same/similar courses in subsequent terms/years. In many cases, instructors might be happy to allow distribution of certain materials. However, doing so without expressed permission is considered a violation of intellectual property rights.

Please alert the instructor if you become aware of intellectual property belonging to others (past or present) circulating, either through the student body or online. The intellectual property rights owner deserves to know (and may have already given their consent).

Please see the <a href="Academic Regulations">Academic Regulations</a> (<a href="https://uwaterloo.ca/academic-calendar/undergraduate-studies/catalog#/policy/SkiPsB9Vp?">https://uwaterloo.ca/academic-calendar/undergraduate-studies/catalog#/policy/SkiPsB9Vp?</a>
<a href="https://www.bccurrent=Assessments%3A%20Academic%20Considerations%20and%20Accommodations&bcGroup=Academic%20Regulations&bcItemType=policies">https://www.bccurrent=Assessments%3A%20Academic%20Considerations%20and%20Accommodations&bcGroup=Academic%20Regulations&bcItemType=policies</a>
section of the Undergraduate Calendar for details. Regardless of the process used to declare an absence, it is the student's responsibility to contact their instructor so that the instructor can determine how to accommodate the missed work.

#### Absences due to religious, creed, and spiritual observances

Beginning Fall 2024, students can submit a <u>Religious Observance Self-Declaration Form in Quest</u>. This form allows the student to register dates of potential conflicts with coursework. Students are expected to record their absence within two weeks of announcement of the due date or scheduled examination date for which academic accommodation is being sought. Submission through Quest will notify your instructors of your absence.

## Absences due to verified illness or extenuating circumstances (e.g., bereavement)

For absences due to illness that are not pandemic-related, students need to obtain a <a href="Verification of Illness Form">Verification of Illness Form</a> (<a href="https://uwaterloo.ca/campus-wellness/health-services/student-medical-clinic/verification-illness-services/">VIF</a>) (VIF) that is completed and signed by a healthcare practitioner. The VIF indicates the determination by a care provider of the degree of incapacitation for a given period. The VIF should then be submitted by the student to the must be uploaded and submitted through the <a href="Verification of Illness or Extenuating Circumstances Process">Verification of Illness or Extenuating Circumstances Process</a> (<a href="https://uwaterloo.ca/arts/undergraduate/student-support/accommodations-illness-or-extenuating-circumstances#submit-other/">Verification of Illness or Extenuating Circumstances Process</a> (<a href="https://uwaterloo.ca/arts/undergraduate/student-support/accommodations-illness-or-extenuating-circumstances#submit-other/">Undergraduate/student-support/accommodations-illness-or-extenuating-circumstances#submit-other/</a>. Submission through the <a href="https://ubsence/absence/definitions/besself/">Absence Verification System</a> (<a href="https://ubsence/https://ubsence/absence/definitions/besself/">https://ubsence/definitions/besself/</a> (<a href="https://ubsence/https://ubsence/definitions/besself/">https://ubsence/https://ubsen

For absences due to extenuating circumstances (e.g., bereavement, serious family illness), students submit a request for absence due to extenuating circumstances (https://uwaterloo.ca/arts/undergraduate/student-support/accommodations-illness-or-extenuating-circumstances#submit-other) to the Absence Verification System (https://vif.uwaterloo.ca/) for processing. Students use the form to submit supporting documentation, which will be considered in a comparable manner to a VIF. Official documentation is necessary before any action can be taken. Examples are plane/train/bus tickets, court documents, police reports, death certificates/obituaries/letters from funeral directors.

#### Self-declared absences due to COVID-related illness

Students can <u>self-declare (https://uwaterloo.ca/quest/help/students/how-do-i/self-declare-absence-undergraduate-students)</u> an absence on Quest due to COVID-related illness or a requirement to self-isolate. A self-declared COVID absence covers up to 10 days and no documentation is required. No in-person course activity is permitted during the period of a self-declared COVID absence. Participation in on-line components is permitted but should not be required if the student notifies the instructor that they are too ill to participate in the online work. Submitting the self-declaration through Quest will notify your instructors of your absence.

### Self-declared short-term absences for any reason

Students can <u>self-declare (https://uwaterloo.ca/quest/help/students/how-do-i/self-declare-absence-undergraduate-students)</u> one short-term absence per term on Quest for any reason. A short-term absence covers two calendar days, and <u>no documentation is required</u>. The student has only one self-declared short-term absence per term and this absence can only be used during the formal lecture period. Self-declared absences cannot be applied to Laboratory (LAB), Clinic (CLN), or Studio (STU) course components. Submitting the self-declaration through Quest will notify your instructors of your absence.

## **Cross-listed courses**

Please note that a cross-listed course will count in all respective averages no matter under which subject code it has been taken. For example, a PHIL/PSCI

cross-list will count in a Philosophy major average, even if the course was taken under the Political Science subject code.

## **Anti-Racism Statement**

The University of Waterloo does not tolerate racism or any other form of discrimination and expects campus community members to contribute to a culture where all members feel safe and valued. Any member of the campus community who has experienced racism or discrimination at the University is encouraged to review available processes for addressing their concerns under Policy 33 – Ethical Behaviour and to seek guidance from the Equity Office via email at <a href="mailto:equity@uwaterloo.ca">equity@uwaterloo.ca</a> (https://uwaterloo.ca/presidents-anti-racism-taskforce/part-report/equity@uwaterloo.ca)\_or through their <a href="mailto:website">website</a> (https://uwaterloo.ca/equity-diversity-inclusion-anti-racism/)\_

## Accommodating religious, creed, and spiritual observances

The University of Waterloo has a duty to accommodate religious, spiritual, and other creed-based beliefs and practices (https://uwaterloo.ca/academic-calendar/undergraduate-studies/catalog#/policy/SkiPsB9Vp) under the Ontario Human Rights Commission (2015) Policy on preventing discrimination based on creed (https://www.ohrc.on.ca/en/policy-preventing-discrimination-based-creed). Students may seek accommodations for missed course components on religious, spiritual, or other creed grounds. Students can complete the Religious Observance Self-Declaration Form in Quest, which will inform their instructors of the potential conflict for certain dates. Students must consult with their instructor(s) within two weeks of the announcement of the due date or scheduled examination date for which academic accommodation is being sought. The self-declaration form for short-term absences may also be used by students requiring an absence of two days or less during the Formal Lecture Period.

## Academic freedom at the University of Waterloo

Policy 33, Ethical Behaviour (https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-33) states, as one of its general principles (Section 1), "The University supports academic freedom for all members of the University community. Academic freedom carries with it the duty to use that freedom in a manner consistent with the scholarly obligation to base teaching and research on an honest and ethical quest for knowledge. In the context of this policy, 'academic freedom' refers to academic activities, including teaching and scholarship, as is articulated in the principles set out in the Memorandum of Agreement between the FAUW and the University of Waterloo, 1998 (Article 6). The academic environment which fosters free debate may from time to time include the presentation or discussion of unpopular opinions or controversial material. Such material shall be dealt with as openly, respectfully and sensitively as possible." This definition is repeated in Policies 70 and 71, and in the Memorandum of Agreement, Section 6

# **University Policy**

Academic integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. [Check the Office of Academic Integrity (https://uwaterloo.ca/academic-integrity/) for more information.]

**Grievance:** A student who believes that a decision affecting some aspect of their university life has been unfair or unreasonable may have grounds for initiating a grievance. Read <u>Policy 70, Student Petitions and Grievances, Section 4 (https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-70). When in doubt, please be certain to contact the department's administrative assistant who will provide further assistance.</u>

**Discipline:** A student is expected to know what constitutes academic integrity to avoid committing an academic offence, and to take responsibility for their actions. [Check the Office of Academic Integrity (https://uwaterloo.ca/academic-integrity/) for more information.] A student who is unsure whether an action constitutes an offence, or who needs help in learning how to avoid offences (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course instructor, academic advisor, or the undergraduate associate dean. For information on categories of offences and types of penalties, students should refer to Policy 71, Student Discipline (https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-71). For typical penalties, check Guidelines for the Assessment of Penalties (https://uwaterloo.ca/secretariat/guidelines/guidelines-assessment-penalties).

Appeals: A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-70) (other than a petition) or Policy 71, Student Discipline (https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-71) may be appealed if there is a ground. A student who believes they have a ground for an appeal should refer to Policy 72, Student Appeals (https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-72).

Note for students with disabilities: AccessAbility Services (https://uwaterloo.ca/accessability-services/)\_, located in Needles Hall, Room 1401, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with AccessAbility Services at the beginning of each academic term.

**Turnitin.com:** Text matching software (Turnitin®) may be used to screen assignments in this course. Turnitin® is used to verify that all materials and sources in assignments are documented. Students' submissions are stored on a U.S. server, therefore students must be given an alternative (e.g., scaffolded assignment or annotated bibliography), if they are concerned about their privacy and/or security. Students will be given due notice, in the first week of the term and/or at the time assignment details are provided, about arrangements and alternatives for the use of Turnitin in this course.

It is the responsibility of the student to notify the instructor if they, in the first week of term or at the time assignment details are provided, wish to submit alternate assignment.

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